

A NOBEL LAUREATE'S Chiddushei Torah

BY MOSHE KOPPEL

A Jew winning the Nobel Prize has become such a common occurrence as to be almost passé. Nevertheless, when a Jew with a long white beard and a large *kippah* wins the Nobel Prize, Orthodox Jews the world over cannot help but be flushed with pride. So it was when, on December 10, Yisrael Aumann received the Nobel Prize in economic sciences from the King of Sweden.

Professor Aumann is indeed, as one might say, *fun unsere* (one of our own). In the 1940s, he attended Mesivta Rabbi Jacob Joseph in New York, where he is said to have been a quiet boy with the reserved manner of a young refugee from Germany. (Today he is a rather gregarious fellow with a wry wit.) Many years after leaving RJJ, he would tell his children about his special fondness for his Gemara teacher, Rav Shmuel Warshavchik, *z"l*. After attending the City College of New York and MIT, Professor Aumann made *aliyah* to Jerusalem in 1956 with his wife, Esther, the daughter of Dr. Falk Schlesinger, the then-director of Shaarei Tzedek Hospital in Jerusalem. He has been a faculty member of the mathe-

matics department at The Hebrew University of Jerusalem ever since.

Professor Aumann's research has focused on an area of mathematics known as game theory. Very broadly speaking, game theory involves the formalization of optimal behavior of participants in "games" in which each participant's costs and profits depend on the behavior of the other participants. Some of the earliest contributors to the field have become legendary figures, including John von Neumann and John Nash. Professor Aumann has made a number of seminal contributions to game theory, any one of which might have been worthy of a Nobel Prize. These include his analyses of the case in which the same players play the same game repeatedly, the case in which the number of players is essentially infinite and the case in which players have only partial knowledge of relevant information.

Professor Aumann has a special interest in the interface of mathematics and Torah. For many years, he (together with Ely Merzbach, a professor at Bar-Ilan University's Department of Mathematics and Computer Science, and this writer) organized bimonthly meetings of leading Torah scholars and scientists in the home of Meir, *z"l*, and

Sara Brachfeld in Motza. For each meeting one of the participants or a specially invited guest would prepare a lecture on a Talmudic *sugya* that involved some element of mathematics or economics. On these occasions, Professor Aumann would display his particular penchant for asking innocent-sounding questions (in a style reminiscent of Peter Falk in the old *Columbo* series) that completely take the wind out of a speaker's sails. On one particularly memorable occasion, one of Israel's leading *posekim* tried to sneak a fuzzy definition of *ona'ah* (usury) past Aumann and a roomful of economists. It was brutal.

Professor Aumann himself has written and lectured on a number of Talmudic topics to which he has brought his expertise to bear, including questions of statistical majority, risk aversion and equitable distribution of limited resources. This last topic, focusing on a *mishnah* in *Ketubot* (93a), is especially central to Professor Aumann's work. An article he wrote on this *mishnah* has been very widely cited in the economics and game theory literature and has brought awareness of rabbinic reasoning to a broad audience outside of the *beit midrash*.

The question Professor Aumann deals with in his article was brought to

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his attention by his son Shlomo, ז"ל, and his article is dedicated to Shlomo's memory. Shlomo himself was an exceptional person. He is fondly remembered by many as a charismatic leader in the Ezra youth movement in Rechavia. He studied at the Yeshivat Hesder of Sha'alvim, where his uncle, Rav Meir Schlesinger, was *rosh yeshivah*. After completing *hesder*, Shlomo spent a year studying at Mirrer Yeshiva before beginning law school at The Hebrew University of Jerusalem. He fell in battle during Operation Peace for Galilee in 1982, leaving behind a pregnant wife and a son.

I will devote the rest of this article to Professor Aumann's treatment of the *mishnah* in *Ketubot* (93a).

The *mishnah* considers a case in which a man has left behind three wives (or, more generally, creditors) to whom he owes 100, 200 and 300, respectively.¹ The estate, however, is inadequate to cover all of the debts, and therefore some mechanism is needed for distributing the limited funds equitably. The *mishnah* offers three examples of what it regards as the proper distribution. If the estate is worth 100, then it is divided equally, that is, each woman receives $33\frac{1}{3}$. If it consists of 300, it is divided proportionally, that is, each woman receives exactly half of her claim. If it consists of 200, then the woman whose claim is 100 receives 50, and the other two women each receive 75.

The *gemara* notes that the *mishnah* reflects the view of Rabbi Natan but that Rabbi Yehudah HaNasi disagrees. Some opinions in the *gemara* further argue that even Rabbi Natan's rules only apply in special cases. Nevertheless, we might reasonably ask if there is some discernible principle that underlies the three cases. On the face of it, either equal distribution (as in the case of an estate worth 100) or propor-

tional distribution (as in the case of an estate worth 300) makes good sense, but it is not clear why one method should be used in one case and another method in the other. Moreover, the case of the estate worth 200 follows neither method and appears altogether mysterious.

Professor Aumann noted that game theory provides the appropriate



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tools for analyzing the three cases of the *mishnah* in order to find some general principle that can be applied to any size estate and any number of claimants with any size claims. In his original paper,² Professor Aumann shows that the solution offered by the *mishnah* uniquely satisfies a particular set of constraints and can best be described in terms of a game-theoretic concept known as the nucleolus. In a later paper, written for an audience knowledgeable in Talmud but not necessarily in mathematics,³ Professor Aumann sidesteps the game-theoretic technicalities that motivated the earlier results and explains the same ideas in terms of well-known Talmudic concepts.

The central idea, as presented in the latter paper, can be easily grasped. There is a well-known principle for dividing a disputed sum between *two* claimants. According to this method,

described at the very beginning of *Bava Metzia*, each claimant first gets the part conceded by the other claimant, and then what remains is divided equally. Thus, in the famous case of one claiming half and the other claiming the whole, the second claimant receives the half conceded by the first, and the remaining half is split. This leaves the first claimant with one-fourth and the second with three-fourths, as indicated in the *mishnah* there. This same principle appears in several other places in the Talmud.

The question that arises, is can this principle be generalized to more than two claimants? Professor Aumann notes a remarkable fact about the three cases in the *mishnah* in *Ketubot*. In each case, if one isolates any two of the three women, the total that the *mishnah* gives the two collectively is divided between them according to the two-claimant principle of *Bava Metzia*.

Consider, for example, the difficult case of the estate worth 200 and the women who claim 100 and 200, respectively. The *mishnah* gives the two women 125 collectively (50 and 75, respectively). Of this 125, the first concedes 25 (she only claims 100), and the second concedes nothing (her claim is greater than the total), so the second gets the conceded 25, and the remaining 100 is divided equally. This leaves the first with 50 and the second with 75, exactly as stated in the *mishnah*. The same holds for all other pairs of women in all the cases discussed in the *mishnah*.

Professor Aumann concludes that the *mishnah* in *Ketubot* is a generalization of the two-claimant principle in *Bava Metzia*. Most remarkably, Professor Aumann proves that this generalization is unique. No solution to the three-claimant problem of *Ketubot*, other than the one offered, would be consistent in this manner with the

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two-claimant solution offered in *Bava Metz'ia*. In fact, there is only one such solution—and a constructive method for computing it—for any size estate and any number of women with any size claims.

Professor Aumann's generalization of the *mishnah's* problem is by no means the only possible one,⁴ but it serves as an outstanding example of the use of modern mathematical tools in the service of Torah study. Just as Professor Aumann's receiving the Nobel Prize was a great *kiddush Hashem*, his papers on this problem have created a *kiddush Hashem* by drawing the attention of the community of scholars to remarkable rabbinic insights that are only now being rediscovered. **IA**

Notes

1. Though the *mishnah* does not explicitly indicate a particular currency, it appears to be referring to silver dinars. In any case, the choice of currency is irrelevant to the analysis.

2. R.J. Aumann and M. Maschler, "Game Theoretic Analysis of a Bankruptcy Problem from the Talmud," *Journal of Economic Theory* 36, no. 2 (1985): 195-213.

3. "On the Matter of the Man with Three Wives" (Hebrew), *Moriah* 22, nos. 3 and 4 (Tevet 5759/January 1999): 98-107.

4. See, for example, Robert Brody, "A Talmudic Principle of Distribution" (Hebrew), *Higayon* 1 (5749/1989): 95-101.